

GJ is standing 21 feet from HJ, who is 6 feet tall. GJ throws a football at 30 feet per second in HJ's direction, SCORE: ____ / 25 PTS at an angle of 67.38° with the horizontal, from an initial height of 5 feet.

NOTE: $\sin 67.38^\circ = \frac{12}{13}$ and $\cos 67.38^\circ = \frac{5}{13}$

[a] Write parametric equations for the position of the football.

$$x = (30 \cos 67.38^\circ)t = \frac{150}{13}t$$
$$y = 5 + (30 \sin 67.38^\circ)t - 16t^2 = 5 + \frac{360}{13}t - 16t^2$$

[b] Does the football hit HJ, go over HJ's head, or hit the ground before reaching HJ?

$$x = 21$$
$$\frac{150}{13}t = 21$$
$$t = 1.82$$
$$y = 5 + \frac{360}{13}(1.82) - 16(1.82)^2$$
$$y = 2.4016$$
$$0 < 2.4016 < 6$$

THE FOOTBALL HITS HJ

IJ spent 13 hours studying the first week of the quarter. Each week afterwards, IJ's study time was 9% more than the previous week. If the quarter was 12 weeks long, how much time did IJ study over the entire quarter? SCORE: ____ / 10 PTS

$$13 + 13(1.09) + 13(1.09)^2 + \dots + 13(1.09)^{11}$$

GEOMETRIC, $r = 1.09$

$$= \frac{13(1.09^{12} - 1)}{1.09 - 1} = 261.83 \text{ hours}$$

Find the sum of the series $249 + 241 + 233 + 225 + 217 + \dots - 375$.

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ARITHMETIC, $d = -8$

$$249 - 8(n-1) = -375$$
$$-8(n-1) = -624$$
$$n-1 = 78$$
$$n = 79$$
$$S_n = \frac{1}{2}(79)(249 + -375) = -4977$$

Using mathematical induction, prove that $\sum_{i=1}^n 4^{i+1} = \frac{4^{n+2} - 16}{3}$ for all positive integers n .

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Do NOT use the finite geometric series formula in your proof.

BASIS STEP: WHEN $n=1$, $\sum_{i=1}^1 4^{i+1} = 4^2 = 16$

$$\frac{4^3 - 16}{3} = \frac{48}{3} = 16$$

INDUCTIVE STEP:

ASSUME $\sum_{i=1}^k 4^{i+1} = \frac{4^{k+2} - 16}{3}$ FOR SOME ARBITRARY BUT PARTICULAR INTEGER k

$$\begin{aligned} \sum_{i=1}^{k+1} 4^{i+1} &= \sum_{i=1}^k 4^{i+1} + 4^{k+2} \\ &= \frac{4^{k+2} - 16}{3} + 4^{k+2} \\ &= \frac{4^{k+2} - 16 + 3 \cdot 4^{k+2}}{3} \end{aligned} \quad \begin{aligned} &= \frac{4 \cdot 4^{k+2} - 16}{3} \\ &= \frac{4^{k+3} - 16}{3} \\ &= \frac{4^{(k+1)+2} - 16}{3} \end{aligned}$$

BY MI,
 $\sum_{i=1}^n 4^{i+1} = \frac{4^{n+2} - 16}{3}$
FOR ALL POSITIVE
INTEGERS n

Describe the difference between the curves with parametric equations $x = \sin t$ and $x = -t^2$, $y = \cos^2 t$ and $y = 1 - t^4$.

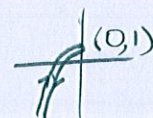
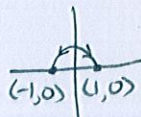
SCORE: ____ / 10 PTS

Discuss the rectangular equation(s) of the graphs, as well as the orientation and portion of the graph corresponding to the parametric equations.

BOTH CURVES CORRESPOND TO $y = 1 - x^2$

AS t GOES FROM $-\infty$ TO ∞

$x = \sin t$ OSCILLATES BETWEEN -1 AND 1 BUT $x = -t^2$ GOES FROM 0 TO $-\infty$



Use sigma notation to write the series $\frac{32}{6} - \frac{80}{24} + \frac{200}{120} - \frac{500}{720} + \dots - \frac{3125}{40320}$.

GEOMETRIC,
 $r = -2.5$

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$$\begin{array}{ccccccc} & \nearrow & \nearrow & \nearrow & & & \\ & \times 4 & \times 5 & \times 6 & & & \\ 3! & 4! & 5! & 6! & & & 8! \end{array}$$

$$\sum_{n=1}^6 \frac{32(-2.5)^{n-1}}{(n+2)!} \quad \text{or} \quad \sum_{n=3}^8 \frac{32(-2.5)^{n-3}}{n!} \quad \text{or} \quad \sum_{n=0}^5 \frac{32(-2.5)^n}{(n+3)!}$$

Consider the expression $(13x^7 - 17x^3)^{27}$.

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- [a] Write the expansion of the expression using sigma notation. **Your answer may use ! but not ${}_nC_r$ notation.**

Simplify all exponents.

$$\sum_{r=0}^{27} {}_{27}C_r (13x^7)^{27-r} (-17x^3)^r = \sum_{r=0}^{27} \frac{27!}{r!(27-r)!} 13^{27-r} (-17)^r x^{7(27-r)+3r}$$

$$= \sum_{r=0}^{27} \frac{27!}{r!(27-r)!} 13^{27-r} (-17)^r x^{189-4r}$$

- [b] Find the coefficient of x^{129} in the expansion. **Your answer may use ! but not ${}_nC_r$ notation.**

$$189 - 4r = 129$$

$$-4r = -60$$

$$r = 15$$

$$\frac{27!}{15!12!} 13^{12} (-17)^{15} = -\frac{27!}{15!12!} 13^{12} 17^{15}$$

Simplify $\frac{(7n-4)!}{(7n-1)!}$.

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$$\frac{(7n-4)!}{(7n-1)(7n-2)(7n-3)(7n-4)!} = \frac{1}{(7n-1)(7n-2)(7n-3)}$$

Eliminate the parameter to find rectangular equations corresponding to the parametric equations

$$x = \frac{t}{2-t}$$

$$y = \frac{t+1}{t-3}$$

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For your final answer, write y as a simplified function of x .

$$x(2-t) = t$$

$$2x - xt = t$$

$$2x = t + xt$$

$$2x = t(1+x)$$

$$t = \frac{2x}{1+x}$$

$$y = \frac{t+1}{t-3}$$

$$= \frac{\frac{2x}{1+x} + 1}{\frac{2x}{1+x} - 3}$$

$$\frac{2x}{1+x} - 3$$

$$= \frac{2x + (1+x)}{2x - 3(1+x)} = \frac{3x+1}{-x-3} = -\frac{3x+1}{x+3}$$